Part B

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MARIE SKŁODOWSKA–CURIE ACTIONS

Individual Fellowships (IF)
Call: H2020-MSCA-IF-2014

PART B

“MiLC”

“Monotonicity in Logic and Complexity”

This proposal is to be evaluated as:

[**Standard EF**]

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List of Participants

Participants	Legal Entity Short Name	Academic (tick)	Non-academic (tick)	Country	Dept. / Division / Laboratory	Supervisor	Role of Partner Organisation
Beneficiary							
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1 Excellence

1.1 Quality and credibility of the research/innovation action (level of novelty, appropriate consideration of inter/multidisciplinary and gender aspects)

Monotonicity is a fundamental notion in mathematics and computation. For usual real-valued functions $\mathbb{R} \to \mathbb{R}$ this simply corresponds to the notion that a function is increasing (or decreasing) in its argument, however this can be parametrised by any partially ordered domain and codomain we wish, *e.g.* for functions $\mathbb{R}^m \to \mathbb{R}^n$ by taking the product order on $\mathbb{R}^i$. In computation we deal with binary strings, *i.e.* finite sequences over $\mathbb{B} = \{0, 1\}$, and a *program* typically computes Boolean functions $\mathbb{B}^* \to \mathbb{B}^*$.[1] Restricting to increasing functions over this structure can be seen as prohibiting the use of *negation* in a program;[2] for instance monotone Boolean functions are computed by Boolean circuits without `NOT` gates. The idea of restricting negation scales to other models of computation, and for some important classes of functions the formulation is naturally robust, not depending on the particular model at hand, *e.g.* for the *polynomial-time* functions.[3] Monotone computational problems abound in practice, *e.g.* sorting a string[4] and detecting cliques in graphs,[5] and 'nonuniform' monotone models of computation, such as monotone circuits, have been fundamental objects of study in computational complexity for decades.[6]

MiLC aims to develop *logical* characterisations of monotone complexity classes, via a *proof theoretic* approach. Namely, the project will identify theories of arithmetic whose formally representable functions coincide with certain monotone classes, and also develop fundamental recursion-theoretic programming languages in which to extract the monotone functions themselves. In particular the project focusses on the role of *structural* proof theory, *i.e.* the duplication and erasure of formulae, in controlling monotonicity.

Logic and complexity MiLC fits into the wider area of *Implicit Computational Complexity* (ICC), which characterises complexity classes in a way independent of any particular model of computation. A principal motivation is to have access to programming languages and models of reasoning that certify the efficiency of representable algorithms. Approaches in ICC typically restrict 'structural' or 'logical' features of basic mathematical objects, such as languages and proof systems. For example, a celebrated result is Fagin's theorem: existential second-order logic can express precisely the properties that are in $\mathbf{NP}$.[7] Some of the most important approaches in ICC today proceed via *proof theory*, the subfield of mathematical logic that studies formal proofs and modes of reasoning. The principal idea is to calibrate principles of reasoning with complexity-theoretic features. This has led to many successful lines of work, employing a variety of complementary methods. Some seminal characterisations include:

- Exponential-time hierarchy via formula rank.[8]
- Polynomial and $\mathbf{NC}^i$-$\mathbf{AC}^i$ hierarchies via bounded arithmetic.[9,10]

[1] The order on $\mathbb{B}^* = \bigcup_{i=0}^{\infty} \mathbb{B}^i$ is typically the union of the product orders.
[2] M. Grigni, M. Sipser. "Monotone Complexity". *London Mathematical Society Symposium on Boolean Function Complexity*. New York, NY, USA: Cambridge University Press, 1992.
[3] C. Lautemann, T. Schwentick,, I. A. Stewart. "On Positive P". *IEEE Conference on Computational Complexity '96*. 1996.
[4] K. E. Batcher. "Sorting networks and their applications". *ACM Spring Joint Computer Conference*. 1968.
[5] C. H. Papadimitriou. *Computational complexity*. John Wiley and Sons Ltd., 2003.
[6] A. D. Korshunov. "Monotone Boolean functions". *Russian Mathematical Surveys* 58.5 (2003).
[7] R. Fagin. "Generalized first-order spectra, and polynomial. time recognizable sets". *SIAM-AMS Proceedings* 7 (1974).
[8] D. Leivant. "Calibrating Computational Feasibility by Abstraction Rank". *IEEE Symposium on Logic in Computer Science '02*. 2002.
[9] S. R. Buss. *Bounded arithmetic*. Vol. 1. Studies in Proof Theory. Naples: Bibliopolis, 1986.
[10] P. Clote, G. Takeuti. "First order bounded arithmetic and small boolean circuit complexity classes". *Feasible Mathematics II*. Springer, 1995.

- Polynomial and elementary time via variants of linear logic.[11][12]
- Polynomial hierarchy via applicative theories.[13]

However, until now, the tools available in ICC have not been adequate to capture monotone classes since their intermediate steps do not typically preserve local monotonicity constraints, as we discuss later. On the other hand, very recent work[14] by the experienced researcher has given proof-of-concept for such an approach by combining features of existing interpretations in the setting of *proof complexity*. The purpose of MiLC is to develop these ideas into an ambitious and overarching ICC approach to monotone complexity, calibrating them with established methods and new extraction techniques. In this way, MiLC will provide novel tools for ICC at large and extend its scope to monotone complexity classes.

Monotone classes and their characterisations An important issue when dealing with monotone classes is that the monotone version of a class, say the polynomial-time predicates, $\mathbf{P}$, is not simply the monotone members of $\mathbf{P}$. In fact, due to a celebrated result of Razborov[15] and subsequent improvements and generalisations due to Alon & Boppana[16] and Tardos[17], we know that there are polynomial-time monotone predicates (and hence polynomial-size nonmonotone circuits) for which the only monotone circuits are exponential in size. Hence any meaningful version of monotone $\mathbf{P}$, *i.e.* one where we can efficiently compile to monotone circuits, is separated from $\mathbf{P} \cap \{\text{monotone predicates}\}$.

Nonetheless, several definitions of monotone $\mathbf{P}$ can be formulated by essentially prohibiting the use of negation in known characterisations of $\mathbf{P}$ and, reassuringly, the notion is rather robust: all known definitions coincide[18] and yield polynomial-size monotone circuits, as expected.

MiLC will extend such formulations to other 'feasible' complexity classes and their relatives, due to their importance in circuit complexity and relationships to proof complexity.[19] Specifically we consider the weaker classes $\mathbf{NC}^i$ and $\mathbf{AC}^i$ and also the stronger classes forming the polynomial hierarchy, $\mathbf{PH}$.[20] The nonuniform versions of $\mathbf{NC}^i$, $\mathbf{AC}^i$ and $\mathbf{P}$ are well understood in the monotone setting, via negation-free circuits. MiLC will establish robust recursion-theoretic formulations of the uniform versions of these monotone classes, as well as a monotone version of $\mathbf{PH}$ and its levels.

Witness extraction Given a proof in an appropriate formal theory, there are many ways of extracting programs from it. Perhaps the most popular of these are *realisability*[21] and *Dialectica*[22] interpretations, however both of these first require a 'double negation' translation, or a change of logical basis to the *negative* fragment $\{\bot, \wedge, \Rightarrow, \forall\}$. This destroys local monotonicity in a proof at the level of formulae occurring, and so crucial invariants are not able to control monotonicity of extracted functions.

However a lesser known approach, the *Witness Function Method* (WFM), turns out to be ideal for controlling monotonicity. It was discovered by Buss who used it principally to prove results on the representable functions of bounded arithmetic.[23] It has been particularly successful at delineating hierarchies of feasible complexity classes (*e.g.* $\mathbf{NC}^i$, $\mathbf{AC}^i$, $\mathbf{P}$, $\mathbf{PH}$), and remains the foremost method of conducting witness extraction directly from classical theories. The WFM proceeds as follows:

[11] J.-Y. Girard. "Light linear logic". *Logic and computational complexity*. Springer. 1995.

[12] P. Baillot. "On the expressivity of elementary linear logic: Characterizing Ptime and an exponential time hierarchy". *Information and Computation* 241 (2015).

[13] R. Kahle, I. Oitavem. "Applicative theories for the polynomial hierarchy of time and its levels". *Annals of Pure and Applied Logic* 164.6 (2013).

[14] A. Das. "From Positive and Intuitionistic Bounded Arithmetic to Monotone Proof Complexity". *IEEE Symposium on Logic in Computer Science '16*. Ed. by N. Shankar. In press. IEEE.

[15] A. A. Razborov. "Lower bounds on the monotone complexity of some Boolean functions". *Doklady Akademii Nauk SSSR* 285 (1985).

[16] N. Alon, R. B. Boppana. "The monotone circuit complexity of Boolean functions". *Combinatorica* 7.1 (1987).

[17] E. Tardos. "The gap between monotone and non-monotone circuit complexity is exponential". *Combinatorica* 8.1 (1988).

[18] Lautemann, Schwentick,, Stewart, "On Positive P".

[19] Buss, *Bounded arithmetic*.

[20] Here 'weaker' and 'stronger' refer to the inclusion of classes.

[21] A. S. Troelstra. "Realizability". *Handbook of Proof Theory* (1998).

[22] J. Avigad. "Gödel's functional ("Dialectica") interpretation". *Handbook of Proof Theory* 137 (1998).

[23] Buss, *Bounded arithmetic*.

1. Reduce a proof to *De Morgan* normal form, with formulae written over the basis $\{\bot, \top, \vee, \wedge, \exists, \forall\}$ and negation restricted to atoms.
2. Conduct a *free-cut elimination* on the proof, resulting in a proof whose formulae are restricted to essentially just subformulae of the conclusion, axioms and nonlogical steps.
3. Extract witnesses by an inductive interpretation of the proof. This relies on witnessing invariants with appropriate logical properties induced by those of formulae occurring in a free-cut free proof.

This final point is important: the role of 2 is to ensure that we have access to appropriate witnessing invariants. In particular the logical complexity of witnessing invariants is correlated with that of formulae occurring in a proof, which in turn affects the complexity of extracted functions. The experienced researcher has recently observed that this scales also to the notion of monotonicity:[24] if axioms and nonlogical steps are monotone then 2 yields monotone proofs and so also monotone witnessing invariants.

MiLC will fundamentally rely on 2 to generate monotone invariants of theories. The WFM will be employed as the main method of witness extraction, but will also be calibrated with forms of realisability to admit monotone interpretations of stronger theories with certain structural restrictions.

Sources of nonmonotonicity Nonmonotonicity is essentially induced by structural rules of the underlying logic. In particular, consider the case of right-contraction, or idempotency of disjunction:[25]

$$A \vee A \Rightarrow A \tag{1}$$

A witness of this axiom should be a map from a witness of $A \vee A$ to a witness of A. The former should be a pair (σ, τ) such that either σ or τ witnesses A, depending on the assignment to free variables, say $\vec{a}$. Assuming decidability of the witness predicate, one can proceed by defining a *conditional* function f:

$$f(\sigma, \tau, \vec{a}) = \begin{cases} \sigma & \text{if } \sigma \text{ witnesses } A(\vec{a}) \\ \tau & \text{otherwise} \end{cases}$$

However f is not necessarily monotone in $\vec{a}$; in fact the use of conditionals is precisely where nonmonotonicity occurs in recursion-theoretic schemes. MiLC will investigate two solutions to this problem:

(a) Control negation to construct theories for which the witness predicate is monotone. Now (1) can be witnessed by mapping σ and τ to their least upper bound $\sigma \vee \tau$ in the Boolean lattice.

(b) Control problematic uses of contraction via *substructural* restrictions to the underlying logic.

(a) is inspired by recent work of the experienced researcher on monotone proof complexity, resulting in 'positive' and 'intuitionistic' formulations of bounded arithmetic.[26] MiLC will use these formulations as starting points, but will take on the more ambitious task of relating them to monotone complexity classes for which less is known, due to the lack of robust recursion-theoretic formulations.

For (b) MiLC will develop theories of bounded arithmetic in *linear logic*, to control the presence of contraction, *cf.* (1). Again, recent work by the experienced researcher on free-cut elimination for linear logic[27] provides proof-of-concept that such an approach admits the appropriate witnessing methods.

Objectives and overview of the action The goal of MiLC is to establish fundamental logical tools to reason about monotone complexity, a thus far unaddressed issue in ICC. This will extend the scope of ICC to new settings, as well as providing it with new tools to attack old problems.

I will exploit the established techniques of bounded arithmetic and realisability to construct theories with adequate control over monotonicity, building upon ideas from my own recent work on positive, intuitionistic and linear theories. MiLC will produce robust recursion-theoretic formulations of monotone complexity classes and tight correspondences with these theories via novel methods of extraction.

[24] Das, "From Positive and Intuitionistic Bounded Arithmetic to Monotone Proof Complexity".
[25] The same problem presents in intuitionistic logic in the form of the left-disjunction rule.
[26] Das, "From Positive and Intuitionistic Bounded Arithmetic to Monotone Proof Complexity".
[27] P. Baillot, A. Das. "Free-cut elimination in linear logic and an application to a feasible arithmetic ". CSL '16.

Finally, by way of application, I will frame these results in the setting of *proof complexity*, an area on which I have considerable expertise, and use them to reformulate and improve known bounds. Throughout the action I will be immersed in an environment of excellence in ICC at DIKU, bringing additional expertise and experience to various aspects of the work plan, as I discuss in the sequel.

Research methodology and approach I will develop the aforementioned ideas regarding witness extraction and monotonicity control to achieve a comprehensive ICC approach to monotone complexity. This will be implemented in the framework of bounded arithmetic, where the WFM has had much success and where recent advances suggest an adequate capacity for controlling monotonicity. The investigations induced by (a) and (b) above will translate to individual work packages, **WP2** and **WP3** respectively. In parallel I will construct recursion-theoretic characterisations of monotone classes in which to extract witnesses, forming **WP1**. While this will be a dependency for **WP2** and **WP3**, there will be a natural interplay with these two packages to drive the appropriate characterisations. **WP3** will also somewhat depend on **WP2**, using its proof theoretic constructions as tools for witness extraction. At the conclusion of **WP1**,**WP2** and **WP3** I will turn to applications of this work to proof complexity, forming **WP4**.

This work plan will benefit from considerable local and surrounding expertise including recursion theory (**WP1**; Jakob Simonsen, Jean-Yves Moyen, DIKU), bounded arithmetic and the WFM (**WP2**, **WP3**; Samuel Buss, UCSD), and linear logic and realisability (**WP2**, **WP3**; Thomas Seiller, DIKU).

WP1: Recursion-theoretic characterisations of monotone classes (low risk). I will adapt known characterisations of $\mathbf{NC}^i$, $\mathbf{AC}^i$, $\mathbf{P}$ and $\mathbf{PH}$ to the monotone setting. Specifically, I will consider Cobham style[28] and Bellantoni-Cook style[29] recursion-theoretic characterisations, replacing conditionals with monotone operations, *cf.* (a). Monotone $\mathbf{P}$ will be a natural starting point, being the most well established,[30] after which characterisations of other classes will be guided by parallel work in **WP2** and **WP3**.

WP2: Representable functions of positive and intuitionistic bounded arithmetic (low-medium risk). I will formulate versions of positive and intuitionistic bounded arthmetic, building on recent work,[31] and develop witnessing methods to extract monotone programs in the setting of **WP1**. In the positive setting I will implement the idea of 'joins' witnessing contractions, *cf.* (a). For the intuitionistic setting, I will calibrate the WFM with a form of realisability, extracting to *higher-order* monotone programs. In both settings I will be sensitive to delineating levels of respective hierarchies, *e.g.* $\mathbf{NC}$, $\mathbf{AC}$, $\mathbf{PH}$.

WP3: Bounded arithmetic in linear logic (medium risk). I will develop a *bona fide* version of bounded arithmetic in linear logic, identifying complexity classes with restrictions on connectives, in the spirit of the linear logic approach to ICC. Restrictions to contractive behaviour will be used to control monotonicity, *cf.* (b), leading to *modular* characterisations of monotone classes, where nonmonotone versions may be recovered by inclusion of appropriate structural rules. I will adapt techniques generalising those of **WP2**, exploiting recent work that serves as proof-of-concept of the WFM methodology in linear logic.[32]

WP4: Perspectives on proof complexity (medium-high risk). I will prove correspondences between the theories developed in **WP2** and **WP3** and intuitionistic, monotone and deep inference proof systems,[33] in the spirit of bounded arithmetic.[34] Finally, I will aim to use the setting of arithmetic and the various witnessing techniques to reformulate and improve existing complexity bounds on these systems.

Originality and innovative aspects of the programme An ICC approach to monotone complexity represents an original contribution to the area. Some of the directions proposed in MiLC, such as bounded arithmetic in linear logic and calibrating the WFM with realisability, are new ideas that bring together

[28] A. Cobham. "The Intrinsic Computational Difficulty of Functions". *Logic, Methodology and Philosophy of Science: Proceedings of the 1964 International Congress*. North-Holland Publishing, 1965.

[29] S. Bellantoni, S. Cook. "A new recursion-theoretic characterization of the polytime functions". *Computational complexity* 2.2 (1992).

[30] Lautemann, Schwentick,, Stewart, "On Positive P".

[31] Das, "From Positive and Intuitionistic Bounded Arithmetic to Monotone Proof Complexity".

[32] Baillot, Das, "Free-cut elimination in linear logic and an application to a feasible arithmetic ".

[33] P. Bruscoli, A. Guglielmi. "On the proof complexity of deep inference". *Transactions on Computational Logic* 10.2 (2009).

[34] J. Krajicek. *Bounded arithmetic, propositional logic and complexity theory*. Cambridge University Press, 1995.

different strands of the logic and complexity community. At the same time I have already shown proof-of-concept of some of these ideas in recent work,[35][36] so it is a good time to develop these techniques further in an ambitious project such as this. The observations linking structural rules to monotonicity have not, as far as I know, appeared before in the literature, so MiLC will develop a novel approach from the ground up. It is my hope that the tools developed will also inspire renewed attacks on old problems in ICC, in particular the delineation of elusive hierarchies in the linear logic approach, such as $\mathbf{PH}$.

1.2 Quality and appropriateness of the training and of the two way transfer of knowledge between the researcher and the host

Training programme for the fellow I have 3 years of postdoctoral experience and a strong record of high quality publication at leading venues. The overriding training objective is to give me the opportunity to devote 2 years to a novel and ambitious research programme that will increase my standing and influence in the community. This will significantly enhance my prospects of gaining a tenured position at a leading European institution.

The training programme will develop both scientific and transferable professional skills to strengthen my CV overall. Scientifically, I will learn about various angles of ICC by interacting with researchers at DIKU, via weekly group meetings and one-to-one sessions with Professor Simonsen. An expert in constructive mathematics and ICC, Professor Simonsen has pioneered several distinct approaches, for instance via dynamical systems[37], term rewriting[38][39] and topos theory[40]. He thus has experience of building up novel research in the area and will act as a resourceful mentor and collaborator.

About 6 months into the fellowship I will undertake a short stay at the University of California San Diego (UCSD), where I will collaborate with Professor Samuel Buss, a pioneer of bounded arithmetic and one of the foremost proof theorists in the world. I will take this opportunity to deepen my knowledge on bounded arithmetic and study previous work by Professor Buss on its intuitionistic versions.[41]

I will learn from Professor Simonsen valuable management and supervision skills, which he has demonstrated with aplomb, having supervised 6 PhDs and 7 postdocs. While at UCPH I will take a course in project management and teaching,[42] and I will also supervise a Master's or PhD student during the fellowship. Improving my project management skills will prepare me to lead larger scale projects in the future as I progress to a more senior position, while enhanced teaching and supervision skills will strengthen my CV with crucial experience for a tenured position in academia.

Transfer of knowledge to the host I will bring to DIKU expertise in structural proof theory, select aspects of which are already widely employed in ICC and DIKU, *e.g.* realisability and Curry-Howard interpretations of proofs. In particular I will bring expertise of proof complexity and deep inference, providing fresh perspectives on established dogma in computational interpretations and enlightening the general scientific discussion. I will also bring expertise in bounded arithmetic gained during my short stay at UCSD back to DIKU and the European community at large. I will give guest lectures on these topics, and I will also transfer knowledge to DIKU by supervising a Master's or PhD student there.

[35] Das, "From Positive and Intuitionistic Bounded Arithmetic to Monotone Proof Complexity".

[36] Baillot, Das, "Free-cut elimination in linear logic and an application to a feasible arithmetic ".

[37] J. G. Simonsen. "On the computational complexity of the languages of general symbolic dynamical systems and beta-shifts". *Theoretical Computer Science* 410.47 (2009).

[38] D. de Carvalho, J. G. Simonsen. "An implicit characterization of the polynomial-time decidable sets by cons-free rewriting". *Proceedings of RTA '14*. Springer. 2014.

[39] C. Kop, J. G. Simonsen. "Complexity Hierarchies and Higher-Order Cons-Free Rewriting". Leibniz International Proceedings in Informatics (LIPIcs) 52 (2016). Ed. by D. Kesner, B. Pientka.

[40] J. Frey, J. G. Simonsen. "Toposes for Time Complexity Classes". *Developments in Implicit Computational Complexity '16*.

[41] S. R. Buss. "The polynomial hierarchy and intuitionistic bounded arithmetic". *Structure in Complexity Theory*. 1986.

[42] *Introduction to University Pedagogy*. http://www.ind.ku.dk/english/course_overview/iup/

1.3 Quality of the supervision and of the integration in the team/institution

Qualifications and experience of the supervisors Professor Simonsen is a world expert in ICC, and the theory of computation as a whole, witnessed by his numerous publications on these topics at leading venues, *e.g.* the A-ranked journals *Transactions on Computational Logic* and *Theoretical Computer Science*, as well as 8 papers at the A-ranked *RTA* conference series, including the *Best Paper* award in '04.[43] He was the recipient of the Danish government's *Sapere Aude Elite Research Leader* grant, whence he was principal investigator of the 800,000 € project *Complexity via Logic and Algebra*, 2011-16. He is one of only four members of DIKU with a Habilitation degree, and was recently promoted to a full professor at UCPH. He has supervised 6 PhD students and 7 postdocs, and has been the scientist-in-charge of 5 Marie Skłodowska-Curie Action (MSCA) individual fellowships in recent years,[44] all in topics related to logic and complexity. He thus has an excellent track record in guiding and mentoring young researchers.

Hosting arrangements DIKU was founded in 1970 by the Turing Award winner Peter Naur. It is one of the foremost international centres of ICC research, currently hosting 3 professors, 5 postdocs and 4 PhD students on this and related topics. There are regular seminars and working groups where knowledge can be shared and collaborations formed. During the fellowship I will have weekly meetings with Professor Simonsen to assess scientific progress and address practical matters. This will continue during my short stay at UCSD via video call. We will also be able to discuss informally at any time, being in the same laboratory, and I will collaborate directly with him on certain aspects of this project. I will complete a Career Development Plan with Professor Simonsen early in the fellowship and we will revisit this biannually. The mentoring process will be supported by UCPH's *Performance and Development Review* policy and we will specify in advance the goal of securing a tenured position once the fellowship is over.

1.4 Capacity of the researcher to reach and re-enforce a position of professional maturity in research

I am one of the leading young researchers at the interface between logic and complexity, and am recognised as such in the community. For instance I was an invited speaker at *Logic and Computational Complexity* this year, the foremost outlet for such research, and I served on the program committee of *Developments in Implicit Computational Complexity* in 2015. I received my undergraduate degree in mathematics from the University of Oxford, one of the best in the world, followed by a PhD at the University of Bath under the supervision of Dr Alessio Guglielmi, a leading proof theorist and founder of the rapidly growing area of *deep inference*.[45] I undertook a 3 month placement at Charles University Prague in 2011 to learn about proof complexity, and will spend 2 months at UC Berkeley this autumn as a Visiting Scholar to work on the proof theory of arithmetic and logic in general.

I have an excellent publication record, for which I have worked mostly independently. In particular I published 3 single-authored articles during my PhD, a degree of independence and productivity uncommon for students in my field. 3 of my papers have been accepted to the most prestigious conference of our field, *Logic in Computer Science*, of which 2 were single-authored, and 2 of my other conference articles were invited to special journal issues. While I have some experience supervising undergraduate interns[46] and organising events,[47] these are aspects of my CV that I will improve over the course of this fellowship. The security and resources associated with a MSCA fellowship will help make this possible, and also allow me to continue to improve my publication record.

[43] All rankings are from the *CORE* rankings portal: http://www.core.edu.au/conference-portal.

[44] Lukasz Czajka, Tobias Heindel, Cynthia Kop, Jean-Yves Moyen and Thomas Seiller.

[45] http://alessio.guglielmi.name/res/cos/index.html

[46] In 2012 I supervised an undergraduate intern, Alvin Šipraga, for 10 weeks, and in 2016 I supervised an undergraduate intern, Cameron Calk, for 6 weeks.

[47] In particular, I organised the *Workshop on Efficient and Natural Proof Systems* in 2015.

2 Impact

2.1 Enhancing the potential and future career prospects of the researcher

My main professional goal is to gain a tenured position in Europe within 1-2 years of the fellowship. The exposure to world experts from different parts of ICC will broaden my expertise and improve the quality and dissemination of my work. The experience will train me to formulate pertinent questions and pursue impactful lines of work in the future, *e.g.* by applying techniques developed and knowledge gained during the fellowship to resolve open problems in proof complexity.[48] The project itself will allow me to grow my publication record and market my research to appropriate communities; at least 6 papers will be written, which is a significant output for researchers in my field. Expanding my audience and network of collaborators will not only help me deliver such publications, but also open the door to recommendations from leaders in the field, *e.g.* from Professors Simonsen and Buss. Thus the outputs of both MiLC and my consequent future work will significantly enhance my profile in the community.

Being awarded a MSCA fellowship will display my ability to win funding from prestigious research councils, an important criterion for tenure, and will increase my international experience. At the same time I will learn valuable skills in project management, useful for further funded projects in my career. During the fellowship I will undertake enhanced teaching and supervision opportunities, namely lecturing courses and supervising students, and these will consolidate my overall academic profile.

2.2 Quality of the proposed measures to exploit and disseminate the action results

The results of MiLC will be published in leading international conferences and such as LICS, ICALP, CIE, FOSSACS, CSL and journals such as LMCS, MSCS and JSL, continuing my track record of publication at leading venues. Promotion of the project itself will be achieved by presentations at specialised workshops such as LCC and DICE, as well as regular seminar talks in Denmark, France, UK and USA.

I will also organise a specialised workshop on ICC, emphasising the role of monotonicity via selected invited speakers who have worked on monotone complexity, *e.g.* Albert Atserias, Michelangelo Grigni, Clemens Lautemann, Emil Jeřábek and Michael Sipser. At the same time this will provide an opportunity for different communities to come together, in particular boosting connections American and European researchers. This workshop could be organised to coincide with similar events organised by other MSCA fellows at DIKU, exploiting the shared resources available to organise a world-class event.

I do not expect any issues regarding intellectual property during this project. Rather, all the outlets I mentioned have provisions for making published work open access, and I will take advantage of this in order to maximise the scope of this research. In the unlikely event that issues occur regarding intellectual property, the dedicated *TechTrans* office at UCPH will provide me with the necessary support.

2.3 Quality of the proposed measures to communicate the action activities to different target audiences

Over the course of this fellowship I will speak twice a year at local schools in Denmark and France. I will serve as a Marie Curie Ambassador, delivering special sessions on current research to high-school students and inspiring them to pursue careers in research. MiLC itself exhibits certain aspects which would serve as ideal topics for such students, *e.g.* the notion of monotone functions in partial orders and their fixed points. I already have some experience of such activities during my undergraduate degree and I will be speaking to high school students while at UC Berkeley this autumn.[49]

I will propose a course on ICC at the *European Summer School of Logic, Language and Information* (ESSLLI). This will expose undergraduate, Master's and PhD students to the subject area and allow me to raise the profile of MiLC. I taught at ESSLLI in 2015 so already have experience of proposing, organising

[48] P. Pudlák. "Twelve problems in proof complexity". *International Computer Science Symposium in Russia*. Springer. 2008.
[49] *Berkeley Math Circle*. http://mathcircle.berkeley.edu/

and delivering courses at this school. Furthermore one of my colleagues, Ugo Dal Lago, taught a course on ICC at ESSLLI in 2010, so I could work with him to prepare a successful proposal and course.

Finally I will take part in general public outreach events, namely the *Danish Research Festival*[50] in Copenhagen and the *Nuit des Sciences* in Paris, at least once during the fellowship.

3 Quality and Efficiency of the Implementation

3.1 Overall coherence and effectiveness of the work plan

Referring to the methodology in Section 1.1, MiLC will be divided into 3 stages of dependency delimited by major milestones **MS1** and **MS2**. The 1st stage, consisting of **WP2** and part of **WP1**, will focus on the development of recursion-theoretic characterisations of monotone classes to represent functions of positive and intuitionistic bounded arithmetic; this stage will last around 8 months and culminates in **MS1**. The 2nd stage, consisting of **WP3** and the remainder of **WP1**, will switch focus to the logical setting, developing a *bona fide* formulation of bounded arithmetic in linear logic with a sensitivity to monotonicity considerations, having completed **WP2**; this will overlap slightly with the 1st stage and will last 10-12 months, culminating in **MS2**. Finally the 3rd stage will examine relationships between these theories and proof complexity, exploiting the newly developed tools to attack open problems.

All the information of this section is summarised in a Gantt chart in Figure 1.

Work packages and major deliverables Again referring to the methodology in Section 1.1, the following is a summary of the implementation of various work packages and the major deliverables therein.

Deliv.	Description	Output
WP1.i	Characterisation of monotone $\mathbf{P}$ and $\mathbf{NC}^i$-$\mathbf{AC}^i$ hierarchy.	Workshop
WP1.ii	Characterisation of monotone $\mathbf{PH}$.	Journal[51]
WP1.iii	Higher-order versions of above characterisations.[52]	Conference
WP2.i	Representable functions of positive theories via WFM.	Note
WP2.ii	Calibration of WFM with realisability for intuitionistic theories.	Conference
WP3.i	Bounded arithmetic in linear logic and basic monotone characterisations.	Conference
WP3.ii	Extension of **WP3.i** providing modular account of nonmonotonicity	Preprint
WP4.i	Correspondences between theories and propositional sytems.	Journal
WP4.ii	Improved bounds in proof complexity.	Announcement

Other project monitoring As well as the major milestones, there will be other natural project monitoring points during the fellowship, where risk and trajectory can be reevaluated. After 3 months I will have finished basic recursion-theoretic characterisations of monotone classes $\mathbf{P}$ and $\mathbf{NC}^i$-$\mathbf{AC}^i$ hierarchies. By this point some of the important hurdles and risks should be evident for the remainder of the project. The short stay at UCSD will take place around 6 months into the fellowship, to coincide with the development of work in the middle of **WP2** and the beginning of **WP3**. After 12 months, halfway into **WP3**, I will have a good sense of what will and will not be achievable in the linear logic setting and I will thus turn my focus to achievable results in the time frame and writing up the articles associated with **WP3.i** and **WP3.ii**. I will also submit a workshop presentation on my work on **WP3** thus far at this time. After 15 months, at the end of **WP1**, I will organise a workshop on ICC. I will again reassess the status of **WP3** and look to tie this up so as to begin **WP4**. I will also at this time identify a pertinent supervision topic and recruit a student to begin working with me soon thereafter.

[50] http://forsk.dk/the-danish-research-festival
[51] This will also consolidate the contents of **WP1.i**.
[52] These will be produced as required by **WP2.ii**.

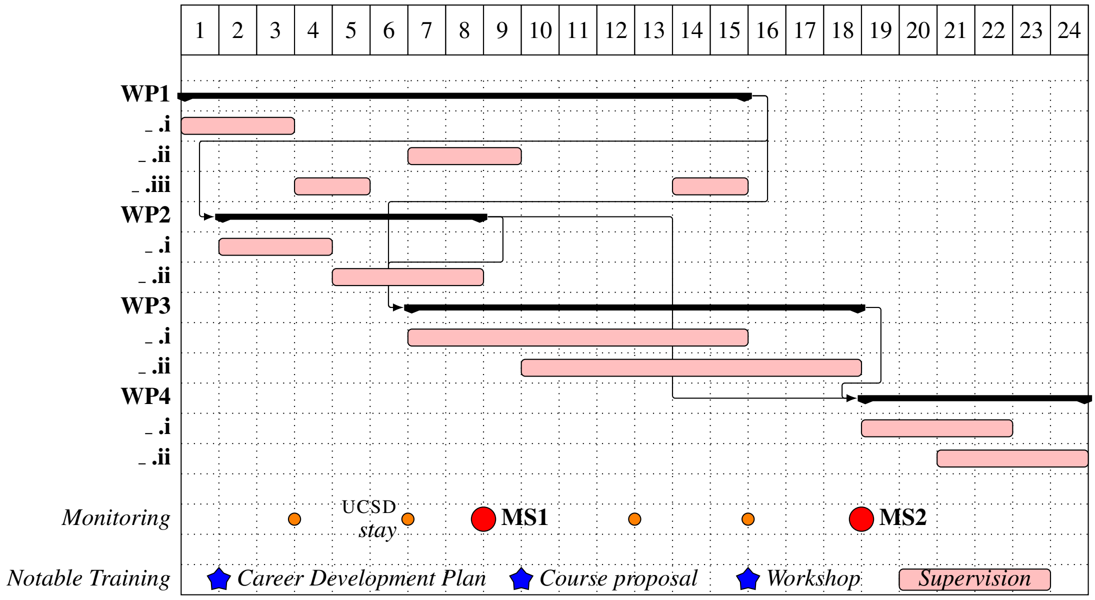

Figure 1: Gantt chart summarising the work plan of this fellowship.

3.2 Appropriateness of the allocation of tasks and resources

The work involved and tasks envisaged are implicit in the major deliverables. The estimated resources required are based on my experience with the ideas and methodology outlined in Section 1.1.

I am proposing to conduct the research for and write 6 articles during the project (3 conference submissions, 2 journal submissions and 1 preprint), and present work at 2 workshops, which is consistent with my current production rate. Much of the research work is concentrated in the 1st half of the project, and this is why the 2nd half will admit more time for training activities, *cf.* Figure 1. Again, this allocation is based on previous experience in teaching, organisation and supervision.

Regarding **WP3**, due to the depth of this research there will be a long stage of theoretical development before major deliverables present. Also, since I will aim for modularity between monotone and nonmonotone characterisations, there will be a degree of calibration and dependency between **WP3.i** and **WP3.ii**. This is why the 2 deliverables will occur at similar times, and also why **WP3.ii** will be made available as a preprint, awaiting the publication of **WP3.i**, rather than directly submitted to a journal.

Professor Simonsen has been scientist-in-charge for 5 MSCA fellows and has been Head of Research at DIKU for 3 years, so he is well experienced in such allocation of time and resources. His workload will be mostly consistent with weekly mentoring and monitoring sessions, however we will also actively collaborate on research when resources are available.

3.3 Appropriateness of the management structure and procedures, including quality management and risk management

Organisation, management structure and progress monitoring to ensure that objectives are reached

I will work on a Career Development Plan early in the fellowship to drive my training objectives. This will be overseen by Professor Simonsen and will be revisited biannually during our weekly supervision meetings. I will also seek advice on career development from Professor Buss during my stay at UCSD.

UCPH has much experience hosting international postdocs and, in particular, MSCA fellows, with approximately 40 hosted by the Faculty of Science over H2020. MiLC will be supported by the Science Research and Innovation unit, who will negotiate the formalities between the Research Executive Agency in Brussels and UCPH. At DIKU the research support unit will assist with administrative and management concerns and the accountancy office will administrate the financial matters of this project, producing regular budget reports and intermediating the expenditure of research and training costs.

My arrival in Copenhagen will be aided by the Housing Foundation and International Staff Mobility units at UCPH, ensuring I am able to quickly find accommodation and settle into the Danish system. UCPH has been awarded the *HR Excellence in Research* title from the EU, attesting to their ability in this regard. Similar resources at UCSD will be made available to me during and before the research stay. In particular, UCSD will support a J1 visa application so that I can legally conduct research work in USA.

Risk analysis and contingency plan As with any ambitious and novel research project, there are various risks in the work plan I have outlined. For the most part, work packages become riskier over time, although monitoring points near the beginning and end of the fellowship are likely to have more impact on the development and framing of the remainder of the project.

WP1.i: Definitions of monotone $\mathbf{NC}^i$, $\mathbf{AC}^i$ not as robust as for monotone $\mathbf{P}$ (low risk). The presence of nonuniform versions of these classes as circuits means that the characterisation can be identified with at least some level of uniformity. This level can be motivated by the needs of **WP2** and **WP3**.

WP1.ii: No obvious candidate for monotone $\mathbf{PH}$ (low-medium risk). Unlike previous classes, we have no nonuniform monotone variant of $\mathbf{PH}$, so it is not obvious that there is some definition at all, regardless of the level of uniformity. This can be addressed by referring to work in WP2 and WP3: if we have a monotone analogue of a theory for $\mathbf{PH}$, then we can at least propose a definition of monotone $\mathbf{PH}$ on these grounds as its provably total functions, represented as a class of appropriate recursive programs.

WP1-4: Mismatch between theories for classes in **WP1** and systems in **WP4** (medium risk). This is entirely possible, since there is no guarantee that correspondences from the nonmonotone world prevail in the monotone world. As evident from the work plan, priority is placed on establishing theories for monotone classes. The mismatch would take away a little the significance of some of the work in **WP4**, but **WP4.i** could still be carried out by adapting theories from **WP2** and **WP3** appropriately.

WP4: No significant applications of WP2 and WP3 to proof complexity (high risk). **WP4.ii** is very ambitious, but at the same time the rewards are high if improvements to the current situation can be found. Given my experience in proof complexity, I think this is a sensible balance of risk and reward. In any case, I expect **WP4.i** to be deliverable in some form, which would itself fill gaps in the literature. Finally, the previous results of **WP1**, **WP2** and **WP3** will be significant enough by themselves; any applications only serve to strengthen the case for their development.

Other minor risks. Unforeseen risks are always a danger, but in the worst case much of **WP2** and **WP3** can be developed independently in a self-contained manner. If a submission is rejected I will act on reviewers comments to revise and resubmit it at the next opportunity, consolidating deliverables if necessary. All work will be made publicly available so there will be no issues of dependency. Finally, if work takes less time than expected then more time can be devoted to **WP4.ii** and training activities.

3.4 Appropriateness of the institutional environment (infrastructure)

UCPH will provide me office space and full access to resources at DIKU. My office will be in the same hallway as Professor Simonsen, making it simple to coordinate and discuss informally. We will furthermore have designated weekly meetings for mentoring, project management and collaboration. I will also attend weekly group meetings and interact with the rest of the group. I will have access to usual office equipment (*e.g.* stationary, whiteboard etc.), a desktop computer and state of the art IT facilities.

I will not require any complex computational power or intellectual property infrastructure. UCPH will support my commitment to open access publishing during this fellowship, and I will have access to their comprehensive physical and electronic libraries to aid my research.

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